Dissimilar Arc Routing Problems

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Abstract

Money collection presents particular problems in terms of effective vehicle routing. Planning the collection or distribution of money for ATMs or parking meters gives rise to two problems: while the total collecting time should be minimized, tours on successive days should be different to prevent robberies.

The combination of these two problems is named as the Dissimilar Routing Problem. When the safes to be collected are located along the streets, it corresponds to an arc routing problem, which we call DARP, and when the money is from ATMs, it corresponds to a vehicle routing problem (DVRP), usually referred to as the peripatetic routing problem. The former problem arises in a Portuguese company in charge of street parking in Lisbon. The firm needs to define tours to collect safes from parking meters, minimizing the total collecting time. To avoid robberies these tours cannot be repeated or somehow anticipated. For this new problem, we present an integer programming (IP) model and develop a matheuristic. Preliminary experiments are provided with data that mimic the real confidential data. Results point to a good performance of the matheuristic, while the smaller instances can be solved to optimality with the IP model and a commercial solver.

Keywords: Arc Routing; Dissimilar Arc Routing; Integer Programming Formulation; Flow Models; Matheuristics; Risk Constrained Cash-in-Transit.

1. Introduction

This paper considers arc routing problems (ARP) were the tours for a specific vehicle must be defined in a weekly based time horizon and, for safety purposes, need to be dissimilar. As in an ARP it starts and finishes its servicing tours in the same point, the depot. The services are associated to the links of the network. The main differences regarding a generic ARP are:

- A weekly time horizon is considered, and the services, as required by the real case in study, are to be performed on a daily basis, which mean that all demand links must be serviced once a day, thus identifying a Rural Postman Problem (RPP) tour per day;
- Dissimilar tours must be defined so as to prevent robberies, and thus we aim to identify one RPP tour per day which, in turn, is somehow dissimilar from remaining tours.

We consider that tours similarity is related with the position each task is served in its tour, and thus, to measure similarity between tours we divide each day into periods. Two tours are fully dissimilar if no task is served during the same period. Of course that, if the number of tasks

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served during the same period in two tours decrease (increase) so thus the dissimilarity (similarity) between them. Imposing the identification of a group of tours, one per day, with no similarities between each pair would be too restrictive. Thus, we opt to relax this imposition, and in accordance to our application, we considered only two types of constraints to avoid similarities. First, by imposing that the service of each arc in two consecutive days is performed in different time periods. Second, by imposing that no more than a given threshold of services can be repeated in the same period in the whole time horizon.

This work was motivated by a case study in a Portuguese company in charge of street parking in Lisbon. The firm needs to define tours to daily collect the safes from parking meters that minimize the total collecting time. To avoid robberies these tours cannot be repeated or somehow anticipated.

The contribution of this paper is fourfold. First, we define and model a new problem, the Dissimilar Arc Routing (DARP), in Section 3. Second, the definition of a dissimilarity measure is proposed for the first time for arc routing cases. Third, in Section 4, we develop a matheuristic, based on three different models here proposed, allowing us the identification of good quality feasible solutions. Fourth, we analyse the application of this methodology to random generated instances, inspired by a real case study involving collection and money transportation, in Section 5.

A literature review, next presented, illustrates the few studies on dissimilarity and routing problems, always focusing on the node routing case.

2. Literature review

Dissimilar tours often appear related to cash-in-transit, security patrol tours, evacuation or even the transportation of hazardous materials. These problems require different approaches, justifying different studies in the literature to tackle them. Additionally, and as far as we know, the works considering dissimilar tours only deal with node routing cases. This paper is thus the first application embedding the dissimilarity of the tours within an arc routing environment. Note that in a node routing case the clients must be visited and reached through dissimilar links. In a parallel arc routing case, the links are the ones to be visited, and the dissimilarity must then be defined on the sequence of repeated links, which, in turn, results in a different and harder problem to solve.

Despite being a critical point with regard to safety, only a few papers address the dissimilarity of the tours, and the VRP version is, as usual, the starting point. (Talarico, et al., 2015) defined and studied the k-dissimilar VRP (kd-VRP), where the similarity between two VRP solutions is defined based on the edges shared between them. The aim is to identify k dissimilar VRP solutions, i.e., tours, starting and ending at a depot node, visiting all clients once, and within the vehicles' capacity. (Talarico, et al., 2015b) defined the Risk-constrained Cash-in-Transit Vehicle Routing Problem (RCTVRP), taking special attention to the risk of being robbed, which they assumed to be proportional both to the amount of cash being carried and to the distance covered by the vehicle carrying the cash. The total risk incurred by a vehicle is, in turn, limited by a risk threshold that can be computed. They also presented metaheuristics and benchmark instances, and further developments on this work gave rise to the recent publication (Talarico, et al., 2016).

An identical problem, also endorsing the node routing case, is referred to as the "Peripatetic" Vehicle Routing Problem (PVRP) introduced by (Krarup, 1975) for the multiple salesman case, and named as the m-Peripatetic Salesman Problem (m-PSP). In this problem no repeated arcs are allowed to visit the clients. Thus, while in the kd–VRP the repetition of arcs is upper limited, the PVRP explicitly forbids repetitions along the planning horizon. These problems also differ on the defined objective. While kd–VRP aims to minimize the worst case travelling cost, the PVRP minimizes the total cost over all periods.

The PVRP may also be applied to network design, as proposed by (De Kort, 1991) whom identify several edges-disjoint cycles to prevent link failure in a network. (De Kort, 1991) and (De Kort, 1993) proposed lower bounds and exact procedures to solve the problem. (De Kort & Volgenant, 1994) generalized the previous studies to tackle a 2-PSP in which each cycle contains each vertex at most once and a penalty is payed for vertices not included in any cycle.

Later on, (Duchenne, et al., 2005) and (Duchenne, et al., 2012) proposed new exact algorithms for the m-PSP to identify disjoint Hamiltonian cycles of minimum total cost. In (Duchenne, et al., 2012) vehicles with limited capacity are also considered.

(Ngueveu, et al., 2010a) and (Ngueveu, et al., 2010b) apply the PVRP to identify patrol tours to security agents, knowing that customers are visited several times within a planning horizon, and no repeated arcs are allowed to reach each client.

(Wolfler-Calvo & Cordone, 2003) studied a security problem where every night, guards must visit all the assigned clients through different tours, amongst other impositions. Alarms can occur, requiring for immediate reaction, i.e. for the redesign of tours in a just-in-time way. An ideal time is identified for each client node in such a way that the times are uniformly distributed through the night. To avoid the tours repetition along the time horizon, time windows are defined around these ideal times, imposing minimum and maximum times between two consecutive inspections.

(Yan, et al., 2012) apply a m-PVRP to reduce operating costs and ensure safety in a cash-in-transit problem. Authors argue that different tours and schedules to enforce safety make it difficult to formulate. The developed model, a multi-commodity network flow, incorporates a similarity defined from both time and space measures for routing and scheduling purposes. Thus, different visit times of the same customer during the planning horizon and different sequences of visited points (space measures) are imposed. In their application, pick-up and delivery services are needed, and thus the amount of money carried by a vehicle along a road is not usually correlated with the number of demand points.

(Michallet, et al., 2014) also deal with cash in transit problems, with the scope to design tours that look “random”, and spread over the time horizon. As the probability of being robbed increases at the vehicles stop (e.g. needed to load or unload an ATM) authors forbid the vehicles arriving out of the clients time windows to avoid waiting times. The problem is named as the periodic VRP with time spread constraints on services (PVRPTS).

The transportation of hazardous materials also demands for dissimilar tours to prevent disasters as well as to not expose always to the same population. Usually, these transports also consider population densities, to try to avoid the use of paths through highly density population areas, making an acceptable trade-off between geographic diversity and performance. (Dadkar, et al., 2008) and (Erkut, et al., 2007) are examples of this type of studies, focusing on the paths diversification as well as some stochastic characteristics. These problems significantly differ from the one here tackled. In fact, different objectives are defined, as minimizing the risk of accidents (e.g. avoiding the use of tunnels) being the solutions’ characteristics also distinct (e.g. the dissimilarity imposed may depend on the population densities and on some pollution aspects in case of accidents). The tours designed are often used repeatedly during some time, and then new and dissimilar tours are found to repeat again for some time, making the dissimilarity issue simpler to handle.

Emergency situations represent other application for dissimilar paths pursing. However, in such cases the dissimilarity is defined to avoid the use of damage paths, resulting in problems significantly distinct from the one in study. A major concern within an emergency case is related to the uncertainty of road conditions after the disaster (earthquakes, hurricanes, chemical explosions, etc.). (Lim & Rhee, 2010), for instance, developed an algorithm to provide alternative paths with overlaps among them.

To sum up, although being more similar to the (kd-VRP), the (RCTVRP) or even the (PVRP) the problem in study is significantly different, and, as far as authors’ knowledge, it is also a new problem. Its challenge comes from the fact that the similarity is here related with the sequences of links traversed, which in turn are harder to identify if compared with a dissimilarity based on links to reach nodes, as in VRP cases. For the first time, we present a new valid model as well as new models to deal within a matheuristic, which is also a novelty.

3. Model for dissimilar mixed arc routing problems

The model here developed is a generalization of the flow based model for the mixed capacitated ARP (MCARP) from (Gouveia, et al., 2010). The problems under study are defined on a mixed graph $(N, A' \cup E)$. Edges in E, characterize narrow two way streets that may be served by only one traversal (zigzag services). Arcs in A' represent either one way or large two way streets that must be served in both directions, in which case the street is modeled with two reverse arcs. The vehicle is parked at a *depot* node, $0 \in N$, from where it starts/ends and its service is performed by only one crew. The depot is far from the service area, and thus no demand arcs are incident into it, and it cannot be used as an intermediate node as well. Node set *N* represents the depot, the street crossings or the dead-end streets.

Two types of links in $A' \cup E$ are distinguished: *demand links* or *tasks*, and *deadheading links* (i.e. links that can be traversed without need of service). All tasks may also be deadheaded for connectivity purposes.

The time horizon is here defined as the set of days $H = \{1, 2, 3, 4, 5\}$. This time horizon is divided into several periods per day.

A *vehicle tour* is a closed walk starting and ending at the depot and representing the vehicle service in a given day, $h \in H$. In the application, each day all the tasks need to be serviced once. A *vehicle service* is a combination of its tours, one for each $h \in H$, that are considered dissimilar.

The *similarity* of two vehicle tours, related to the routine of the services, is defined as the percentage of tasks that is served in the same period. The similarity can be avoided by imposing a threshold limit. The *similarity* of a vehicle service, also named as the *total similarity*, is the sum of the similarity of all pairs of vehicle tours it includes.

We decided to deal with the similarity issue regarding the periods of the day tasks are served. Instead, we might have related it with sequences of tasks or use a combination of both, i.e. periods and sequences. However, we opt to keep it as simple as possible, so it can be studied through a flow base model.

Each link in a tour can be just traversed (deadheaded) or, in the case of a demand link, it can also be served. Each time a link is deadheaded, task or not, a time (or cost) is taken into account.

In what follows, we present the notation used in order to define and model the mixed dissimilar arc routing problems here tackled.

- $\mathrm{G} = (N, A)$ is a directed graph, derived from $(N, A' \cup E)$, by replacing each edge in E by two arcs with opposite directions, i.e. $A = A' \cup \{(i,j),(j,i){:}\ (i,j) \in E\}$ with no repetitions.
- $R \subseteq A$ is the set of arcs in G associated with the tasks, and its cardinality is $|R| = |A_R| + 2|E_R|$, being A_R and E_R the set of arc-tasks and edge-tasks, respectively.
- c_a is the time needed to serve each task $a = (i,j) \in R$.
- d_a is the deadheading time, i.e. the time needed to traverse arc $a = (i,j) \in A$ without serving it.
- $H = \{1, 2, 3, 4, 5\}$ is the time horizon that may represent the days in a working week.
- ℓ is a period index. If we consider one hour periods, we may fix from 9:30 till 12:30 and from 14:00 to 17:00, three morning and three afternoon collecting periods, respectively, numbered from 1 to 6 ($\ell \in \{1, \dots, 6\} = L$).
- W is a large number.

The problem we are modelling basically consists of finding a group of minimum length vehicle tours that are dissimilar in consecutive days. The major differences here included, when compared with (Gouveia, et al., 2010) (an adapted version of this model is presented in the appendix), are the following:

1. Variables are defined with an extra index to identify the periods (ℓ), as the days (index h) represent different tours and thus are related with the multiple tours in the (MCARP) model;
2. New variables and constraints are needed to define different start and ending points per tour to identify the periods;
3. Usual balance and flow constraints on each node must be carefully written as they may be related to a node that will be selected as the starting or the ending node of a period;
4. A minimum number of services per period is imposed to better control the similarity measure;
5. New constraints are added to prevent the similarity of the tours.

Flow based model

For each day $h \in H$ and each period $\ell \in L$, we define:

- $x_{ij}^{\ell h} = \begin{cases} 1 & \text{if } (i,j) \in R \text{ is served during period } \ell \text{ in day } h \\ 0 & \text{otherwise} \end{cases}$
- $u_i^{\ell h} = \begin{cases} 1 & \text{if } i \text{ is the ending point of the tour on period } \ell \text{ in day } h \\ 0 & \text{otherwise} \end{cases}$
- $v_i^{\ell h} = \begin{cases} 1 & \text{if } i \text{ is the starting point of the tour on period } \ell \text{ in day } h \\ 0 & \text{otherwise} \end{cases}$

- $y_{ij}^{\ell h}$ is the number of times that arc $(i,j) \in A$ is deadheaded during period ℓ in day h.
- $f_{ij}^{\ell h}$ is the flow traversing arc $(i,j) \in A$ during period ℓ in day h. It is related to the remaining services in the tour, or in a subtour of it.

The problem to identify a vehicle service in H, minimizing the total routing time, is next detailed.

(M1DAR)

$$\min Z = \sum_{h\in H}\sum_{\ell\in L}\left(\sum_{a\in A} d_a \; y_a^{lh} + \sum_{a\in R} c_a \; x_a^{lh}\right) \tag{1}$$

$$\sum_{j:(i,j)\in A} y_{ij}^{\ell h} + \sum_{j:(i,j)\in R} x_{ij}^{\ell h} - \sum_{j:(j,i)\in A} y_{ji}^{\ell h} - \sum_{j:(j,i)\in R} x_{ji}^{\ell h} = v_i^{\ell h} - u_i^{\ell h} \qquad i \in N\backslash\{0\}; h \in H; \ell \in L \tag{2}$$

$$\sum_{\ell\in L}\left(\sum_{j:(i,j)\in A} y_{ij}^{\ell h} + \sum_{j:(i,j)\in R} x_{ij}^{\ell h} - \sum_{j:(j,i)\in A} y_{ji}^{\ell h} - \sum_{j:(j,i)\in R} x_{ji}^{\ell h}\right) = 0 \qquad i \in N\backslash\{0\}; h \in H \tag{3}$$

$$\sum_{h\in H}\left(\sum_{\ell\in L\backslash\{1\}}\sum_{j:(0,j)\in A} y_{0j}^{\ell h}\right) = 0 \quad \wedge \quad \sum_{j:(0,j)\in A} y_{0j}^{1h} = 1, \quad h \in H \tag{4}$$

$$\sum_{h\in H}\left(\sum_{\ell\in L\backslash\{|L|\}}\sum_{j:(j,0)\in A} y_{j0}^{\ell h}\right) = 0 \quad \wedge \quad \sum_{j:(j,0)\in A} y_{j0}^{|L|h} = 1, \quad h \in H \tag{5}$$

$$\sum_{\ell\in L} x_{ij}^{\ell h} = 1 \qquad a = (i,j) \in A_R\, ; h \in H \tag{6}$$

$$\sum_{\ell\in L}\left(x_{ij}^{\ell h} + x_{ji}^{\ell h}\right) = 1 \qquad a = (i,j) \in E_R; h \in H \tag{7}$$

$$\sum_{a\in R} x_a^{\ell h} \geq \left\lfloor \frac{|A_R| + |E_R|}{|L|} \right\rfloor \qquad \ell \in L;\; h \in H \tag{8}$$

$$\sum_{j:(j,i)\in A} f_{ji}^{1h} - \sum_{j:(i,j)\in A} f_{ij}^{1h} = \sum_{j:(j,i)\in R} x_{ji}^{1h} \qquad i \in N\backslash\{0\}; h \in H \tag{9}$$

$$\sum_{j:(j,i)\in A} f_{ji}^{\ell h} - \sum_{j:(i,j)\in A} f_{ij}^{\ell h} \leq \sum_{j:(j,i)\in R} x_{ji}^{\ell h} + W v_i^{\ell h} \qquad i \in N\backslash\{0\}; \ell \in L\backslash\{1\}; h \in H \tag{10}$$

$$-\sum_{j:(j,i)\in A} f_{ji}^{\ell h} + \sum_{j:(i,j)\in A} f_{ij}^{\ell h} \leq -\sum_{j:(j,i)\in R} x_{ji}^{\ell h} + W v_i^{\ell h} \qquad i \in N\backslash\{0\}; \ell \in L\backslash\{1\}; h \in H \tag{11}$$

$$\sum_{j:(0,j)\in A} f_{0j}^{1h} = \sum_{a\in R} x_a^{1h} \qquad h \in H \tag{12}$$

$$\sum_{j:(i,j)\in A} f_{ij}^{\ell h} \leq \sum_{a\in R} x_a^{\ell h} + W\left(1 - v_i^{\ell h}\right) \qquad i \in N\backslash\{0\}; h \in H;\; \ell \in L\backslash\{1\} \tag{13}$$

$$-\sum_{j:(i,j)\in A} f_{ij}^{\ell h} \leq -\sum_{a\in R} x_a^{\ell h} + W\left(1 - v_i^{\ell h}\right) \qquad i \in N\backslash\{0\}; h \in H;\; \ell \in L\backslash\{1\} \tag{14}$$

$$q_a x_a^{\ell h} \leq f_a^{\ell h} \leq W\left(y_a^{\ell h} + x_a^{\ell h}\right) \qquad a \in R\, ; h \in H;\; \ell \in L \tag{15}$$

$$f_a^{\ell h} \leq W \sum_{\ell\in L} y_a^{\ell h} \qquad a \in A\backslash R\, ; h \in H \tag{16}$$

$$u_i^{\ell h} \leq \sum_{j:(j,i)\in A} y_{ji}^{\ell h} + \sum_{j:(j,i)\in R} x_{ji}^{\ell h} \qquad i \in N\backslash\{0\}; h \in H; \ell \in L \tag{17}$$

$$v_i^{\ell h} \leq \sum_{j:(i,j)\in A} y_{ij}^{\ell h} + \sum_{j:(i,j)\in R} x_{ij}^{\ell h} \qquad i \in N\setminus\{0\}; h \in H; \ell \in L \qquad (18)$$

$$u_i^{\ell h} = v_i^{\ell+1 h} \qquad i \in N\setminus\{0\}; h \in H; \ell \in L\setminus\{|L|\} \qquad (19)$$

$$\sum_{i\in N\setminus\{0\}} u_i^{\ell h} = 1 \qquad h \in H; \ell \in L\setminus\{|L|\} \qquad (20)$$

$$\sum_{i\in N\setminus\{0\}} v_i^{\ell h} = 1 \qquad h \in H; \ell \in L\setminus\{1\} \qquad (21)$$

$$x_{ij}^{\ell h} + x_{ij}^{\ell h+1} \leq 1 \qquad (i,j) \in A_R; h \in H\setminus\{|H|\}; \ell \in L \qquad (22)$$

$$x_{ij}^{\ell h} + x_{ji}^{\ell h} + x_{ij}^{\ell h+1} + x_{ji}^{\ell h+1} \leq 1 \qquad (i,j) \in E_R\, ; h \in H\setminus\{|H|\};\ \ell \in L \qquad (23)$$

$$x_{ij}^{\ell h} \in \{0,1\} \qquad (i,j) \in R; h \in H; \ell \in L \qquad (24)$$

$$f_{ij}^{\ell h} \geq 0 \qquad (i,j) \in A; h \in H; \ell \in L \qquad (25)$$

$$y_{ij}^{\ell h} \geq 0,\ \text{integer} \qquad (i,j) \in A; h \in H; \ell \in L \qquad (26)$$

Conditions (2) and (3) imply the continuity of the vehicle tours at each node, considering three different types of nodes: starting, ending or intermediate; (4) and (5) fix the depot as the starting point of the first period, $\ell = 1$, and ending point of the last period, $\ell = |L|$, each day; the service of each arc and edge, by only one vehicle, is guaranteed by (6) and (7), respectively; (8) imposes a minimum number of services per period, needed to balance periods to avoid situations as the one illustrated in example 1; (9) are flow conservation constraints for the first period, while (10) and (11) represent these constraints for the remaining periods; these, together with the linking constraints (15) and (16) force the connectivity of the vehicle tours. Constraints (12)–(14) define the flow per period and per day; (17) and (18) ensure that the vehicle may use a node as an ending or starting point of a period only if it is traversed by the vehicle during the same period; (19) relates the ending of a period with the beginning of the next period, each day, while (20) and (21) guarantee that only one node may be used as a starting/ending point, per period and per day. Constraints (22) and (23) are used to impose the services dissimilarity. Variable domains are settled in (24) –(26).

Example 1: consider the network with eight edge tasks and two deadheaded links connecting the depot, node 0, depicted in Figure 1. The two feasible tours for two consecutive days, starting and ending at the depot, have no minimum services per period imposed and are:

$$\text{Day1: } \{\underbrace{(0,1,2,3,4,5,3,2,5)}_{\text{period 1}}, \underbrace{(5,6,0)}_{\text{period 2}}\} \quad \text{Day2: } \{\underbrace{(0,6,5)}_{\text{period 1}}, \underbrace{(5,3,4,5,2,3,2,1,0)}_{\text{period 2}}\}.$$

Thus, has no services are repeated during the same period, the tours are considered inappropriately dissimilar. However, we may see that tasks (3,4) and (4,5) are served in exactly the same order (fourth and fifth). If a minimum number of $\left\lfloor \frac{8}{4} \right\rfloor = 4$ services per period is imposed, node 4 will be the ending of period one, and the similarity is detected.

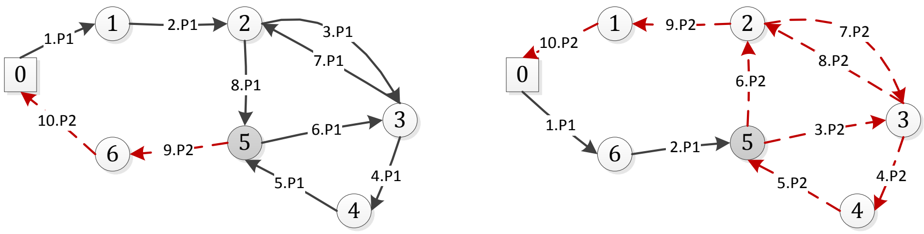


Figure 1: effect of no balancing tours.
Legend: #.P% - # represents the order in the tour and the period %; and the shaded node the node where the first period ends.

Although giving rise to an undesirable increase in the number of constraints, we note that (22) and (23) may be generalized to consider more than two consecutive days. Without this generalization we may get solutions where day $h + 2$ is a replica of day h, and so on, which may represent a model handicap.

Instead, to bound repetitions in the same period all over the time horizon, we may consider the alternative set of constraints:

$$\sum_{h \in H} x_a^{\ell h} \leq M, \quad a \in R, \ell \in L \qquad (27),$$

where $M \geq 1$, and $M = 1$ if no repetitions are allowed.

As referred to above, this would be too restrictive. We thus opt to consider the simpler version, i.e. including only the constraints that avoid similar tours on two consecutive days. More general situations are elaborated through a matheuristic we developed and next detail.

4. Matheuristic

Leaving aside, for now, the similarity issue, this matheuristic starts by generating a pool of feasible tours. Model (M1DAR) is thus applied considering only one day ($h = 1$) as well as different objective functions. With the pool of feasible solutions three models were developed to generate different feasible solutions, regarding the similarity issue. These models aim to select $|H|$ tours (the number of days) that may be consider dissimilar so they can be used in a real case. The matheuristic is next detailed.

Matheuristic:

1. Use model (M1DAR) with $h = 1$, thus without constraints (22) and (23), to identify several feasible tours. Alternative ways to generate the tours.

   Solve (M1DAR) with $h = 1$ and a time limit of 3h;

   Add to the pool of feasible tours, FT, feasible solutions provided by CPLEX;

   Repeat

   i. In FT, fix the service of a task on a period that it is not yet used in the current pool, and run once more model (M1DAR) with $h = 1$, within a three hours cpu time limit;

ii. Add to the pool of feasible tours, FT, all the feasible solutions provided by CPLEX, if any;

Until (all tasks are tried to be serviced in every periods);

iii. Use different objective functions, as e.g. the minimization of deadheading traversals and repeat the above procedure;

2. Use one of the three models: MRHμ; MRμ or MRS, next defined, to identify |H| tours, one per day, that:

   i. minimizes the total time to collect safes not repeating a fixing percentage of tasks in each pair of tours in two successive days – model MRHμ;

   ii. minimizes the total routing time to collect the safes, within a given maximum similarity between any pair of tours – model MRμ;

   iii. minimizes the maximum similarity – model MRS.

The three models referred to in step 2 are next defined.

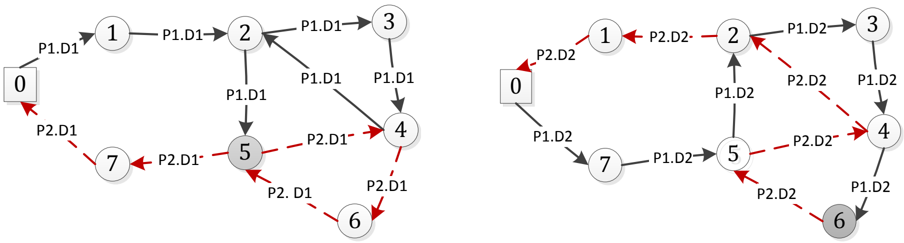
Let: FT be a group of vehicle tours; C_r be the total routing time of tour $r \in FT$; S_{rt} the similarity between tours $r \in FT$ and $t \in FT$; and μ the maximum similarity allowed.

The *similarity index* is computed as: $S_{rt} = \dfrac{\text{number of tasks served during the same period in tours } r \text{ and } t}{\text{total number of tasks}}$ (28).

<u>Example 2:</u> Let us consider the network with nine edge tasks and two deadheaded links connecting the depot, node 0, depicted in Figure 2, and two feasible tours for two consecutive days, starting and ending at the depot:

Day1: $\{\underbrace{(0,1,\overline{2,3,4},\overline{2,5})}_{\text{period 1}},\underbrace{(\overline{5,4},\overline{6,5},7,0)}_{\text{period 2}}\}$ Day2: $\{\underbrace{(0,7,\overline{5,2,3,4},6)}_{\text{period 1}},\underbrace{(\overline{6,5,4},2,1,0)}_{\text{period 2}}\}$.

These tours repeat the periods for servicing tasks (2,3), (3,4), (2,5), in period 1, and tasks (5,4) and (6,5) in period 2. So, the similarity index is $\frac{5}{9}$.



(a) tour for Day1 **(b)** tour for Day2

Figure 2: similarity of two tours.

Legend: P#.D% represents the period # in day %, and the shaded node the node where the first period ends.

Model MRH

The variables are:

- $g_r^h = \begin{cases} 1 & \text{if } r \in FT \text{ is selected for day } h \in H \\ 0 & \text{otherwise} \end{cases}$

and the model to identify the best vehicle service, i.e. the best tours per time horizon minimizing the total routing time is:

(MRHμ)

$$\min \sum_{h \in H} \sum_{r \in FT} C_r \, g_r^h \qquad (29)$$

$$\begin{cases} S_{rt}\left(g_r^h + g_t^{h+1} - 1\right) \leq \mu & r,t \in FT, \;\; h \in H \backslash \{|H|\} \qquad (30) \\ \sum_{r \in FT} g_r^h = 1 & h \in H \qquad (31) \\ g_r^h \in \{0,1\} & r \in FT, \;\; h \in H \qquad (32) \end{cases}$$

Within the minimization of the total routing time objective (29), the aim is to choose one tour per day (31), not allowing tours on two successive days with a similarity index greater than μ (30).

Note that if $\mu = 0$ the solutions provided with this model can fairly be compared with the ones generated by (M1DAR). In fact, both models avoid similar tours not allowing tasks services during the same period in two consecutive days. Thus, if (M1DAR) optimal tours are in the pool FT, the optimal values for both models, (MRH0) and (M1DAR), coincide.

Computational tests are also performed for $\mu = 0.1$ and for $\mu = 0.3$, being so less restrictive regarding the dissimilarity.

As before, instead to prevent the assignment of similar tours in only two consecutive days, we may consider the extra set of constraints:

$$\sum_{h \in H} S_{rt}\left(g_r^h + g_t^h - 1\right) \leq \beta, \quad r,t \in FT \qquad (33),$$

with β a fixed value.

The total similarity, TS, of the tours generated with a model can be computed if (28) is applied to all the pairs of the chosen tours. Thus, and considering a feasible solution of (MRHμ), the total similarity is:

$$TS = \sum_{h=1}^{|H|-1} \sum_{k=h+1}^{|H|} \sum_{r,t \in FT} S_{rt} \, g_r^h \, g_t^k \qquad (34).$$

Next proposition is used to establish the upper bound limits for TS in the above model.

Proposition 1: Any feasible solution of (MRHμ) has a total similarity bounded by:

$$TS \leq \begin{cases} \frac{1}{4}\left[(1+\mu)|H|^2 - 2|H|\right] & \text{if } |H| \text{ is even} \\ \frac{1}{4}\left[(1+\mu)|H|^2 - 2|H| + 1 - \mu\right] & \text{otherwise} \end{cases} \qquad (35).$$

Proof: Let H_1 and H_2 be subsets of odd and even indexes, respectively. Thus, H_1 and H_2 define a partition of H.

$$TS = \sum_{h=1}^{|H|-1} \sum_{k=h+1}^{|H|} \sum_{r,t \in FT} S_{rt} \, g_r^h \, g_t^k =$$

$$= \sum_{\substack{h,k\in H_1 \\ h<k}} \sum_{r,t\in FT} S_{rt}\, g_r^h\, g_t^k + \sum_{\substack{h,k\in H_2 \\ h<k}} \sum_{r,t\in FT} S_{rt}\, g_r^h\, g_t^k + \sum_{h\in H_2} \sum_{k\in H_1} \sum_{r,t\in FT} S_{rt}\, g_r^h\, g_t^k$$

From constraints (31), at most $|H|$ variables g_r^h are nonzero. Note also that, as similarity constraints are only imposed to two consecutive days, not allowing tours with similarities greater than μ, the remaining tours worst case have similarity one, $S_{rt} = 1$. Thus,

$TS \le (|H_1| - 1)\frac{|H_1|}{2} + (|H_2| - 1)\frac{|H_2|}{2} + \mu\, |H_1||H_2|$.

Now, if $|H|$ is odd, $|H_1| = \frac{|H|+1}{2}$ and $|H_2| = \frac{|H|-1}{2}$; otherwise, $|H_1| = |H_2| = \frac{|H|}{2}$, and the result follows. ■

Corollary: Any feasible solution of (M1DAR) has a maximum total similarity given by:

$$TS \le \begin{cases} \frac{1}{4}(|H|^2 - 2|H|) & \text{if } |H| \text{ is even} \\ \frac{1}{4}(|H| - 1)^2 & \text{otherwise} \end{cases} \qquad (36).$$

Proof: First note that (36) is (35) with $\mu = 0$, and that the total similarity, TS, of a feasible solution of (M1DAR) can be computed by:

$$TS = \frac{1}{|A_R \cup E_R|} \sum_{k=h+2}^{|H|} \sum_{h=1}^{|H|-2} \sum_{\ell\in L} \sum_{a\in A_R\cup E_R} x_a^{\ell h}\, x_a^{\ell k} \qquad (37),$$

where, for this purpose, for every edge task $a = (i,j) \in E_R$, $x_a^{\ell h} = x_{ij}^{\ell h} + x_{ji}^{\ell h}$.

Thus, to prove the corollary it is enough to show that (37) is equivalent to (34) if $\mu = 0$, which, in turn, is equivalent to:

$$\sum_{r,t\in FT} S_{rt}\, g_r^h\, g_t^k = \frac{1}{|A_R \cup E_R|} \sum_{r,t\in FT} \begin{bmatrix} \text{number of tasks served during the} \\ \text{same period in tours } r \text{ and } t \end{bmatrix} g_r^h\, g_t^k.$$

As $\mu = 0$, the selection of tours for consecutive days can only consider pairs of tours with no tasks served during the same period. Remaining pairs of tours may repeat services in the same periods. Thus, the above summation counting the number of tasks served during the same period in any pair of selected tours r and t can be written as:

$$\sum_{a\in A_R\cup E_R} x_a^{\ell h}\, x_a^{\ell k},$$

since these products of x-variables assume value one iff task a is served in the same period in days h (with a corresponding tour, say r) and k (with a corresponding tour, say t). ■

Model MR

Let now simplify the model, and define the variables without the identification of the days, as:

- $g_r = \begin{cases} 1 & \text{if } r \in FT \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$

and the model to generate the best vehicle service, i.e. the best tours minimizing the total routing time is:

(MRμ)

$$\min \sum_{r \in FR} C_r \, g_r \tag{38}$$

$$\begin{cases} S_{rt} \left(g_r + g_t - 1 \right) \leq \mu \qquad \forall r, t \in FT & (39) \\ \sum_{r \in FT} g_r = |H| & (40) \\ g_r \in \{0,1\} \quad \forall r \in FT & (41) \end{cases}$$

Within the minimization of the total routing time objective (38), the aim is to choose as many tours as the number of days (40), with a predefined upper bound (μ) on the similarity between any pair of chosen tours (39).

Note that, in this model, μ indicates the maximum percentage of tasks that can be served in the same period for any pair of tours in a vehicle service, and not only for two consecutive days, as in model (MRHμ),

Observe that in example 2 tours are incompatible for this problem if $\mu = 0.3$, as in nine tasks, no more than three services can repeat the period, and we have five repetitions.

The total similarity, TS, of a feasible solution of (MRμ) is:

$$TS = \sum_{r,t \in FT} S_{rt} \, g_r \, g_t \tag{42}$$

Proposition 2: The total similarity of any feasible solution of (MRμ) is bounded by: $TS \leq \mu \, C_2^{|H|}$.

Proof: Observe that if $g_r = g_t = 1$, then $g_r + g_t - 1 = g_r \, g_t = 1$. Otherwise, if $g_r = 0$ or $g_t = 0$ then $g_r + g_t - 1 = 0, -1$ and $g_r \, g_t = 0$. From the definition, in (42) we have:

$$TS = \sum_{r,t \in FT} S_{rt} \, g_r \, g_t \leq \mu \sum_{r,t \in FT} g_r \, g_t \underbrace{=}_{\substack{\text{from (40) exactly} \\ |H| \text{ variables } g_r \\ \text{are non zero}}} \mu \, C_2^{|H|}. \qquad \blacksquare$$

Model MRS

To minimize the maximum similarity, defined as variable $SMax$, we solve the following model. This model is applied to give us an idea about the values for μ parameter in (MRμ) that allow the identification of feasible solutions.

(MRS)

$$\min \ SMax \tag{43}$$

$$\begin{cases} S_{rt}\left(g_r + g_t - 1\right) \le SMax \quad \forall r,t \in FT & (44)\\ \sum_{r\in FT} g_r = |H| & (45)\\ g_r \in \{0,1\} \quad \forall r \in FT & (46)\\ SMax \ge 0 & (47)\end{cases}$$

The maximum similarity is defined in (44) through the tours assigned and as a positive variable (47). Constraints (45) ensure that are assigned as many tours as the number of days.

The total similarity, TS, of a feasible solution of (MRS) can be computed with equation (42).

Proposition 3: The maximum total similarity of any feasible solution of (MRS) is computed by: $TS \le C_2^{|H|}$.

Proof: As in the last prove, and from the definition in (42) we have:

$$TS = \sum_{r,t\in FT} S_{rt}\, g_r \ g_t \underbrace{\le}_{(44)} SMax \sum_{r,t\in FT} g_r \ g_t \underbrace{=}_{\substack{\text{from (45) exactly}\\ |H| \text{ variables } g_r\\ \text{are non zero}}} C_2^{|H|}. \qquad \blacksquare$$

Observe that S_{rt} maximum (value one), is achieved whenever tours r and t every task is assigned to the same period. Of course this is true if r and t represent exactly the same tour, but it is far from being the only situation, as two tours may be different and serve all the tasks in the same periods.

5. Computational results

The proposed models are evaluated over some newly generated instances, as the problems are also new. The computational results were obtained using CPLEX 12.6.0.0, with default settings, in a computer with 2 AMD Opteron 6172 processors (24 cores) at 2.1GHz and with 64 GB RAM. A time limit of three hours was established, each time the CPLEX was used. When an integer program is being solved and the time limit is reached before an optimal solution is proved to be found, CPLEX provides the best bounds that are computed taking into account all the live nodes of the branch-and-cut tree. Such bounds are used to evaluate the procedures.

5.1. Data instances

Twelve instances (**ex1** to **ex12**) were generated to assess the performance of the models, with dimensions varying between 11 to 50 nodes and 34 to 129 links. The graphs are based on real street networks, while deadheading and service times were randomly generated. The relevant characteristics of these instances are depicted in Table 1. Different number of days are also considered, namely, $|H| = 3, 4, 5$.

Name	$|V|$	$|A|$	$|A_R|$	$|E|$	$|E_R|$
ex1	11	33	11	1	1
ex2	16	34	14	2	2
ex3	19	40	18	2	2
ex4	21	47	19	3	3
ex5	21	63	21	2	2
ex6	24	63	25	2	2
ex7	25	54	24	10	10
ex8	35	80	35	16	16
ex9	40	65	45	12	12
ex10	34	77	29	18	18
ex11	45	98	42	32	32
ex12	50	111	66	18	18

Table 1: characteristics of the instances.

5.2. Results

In step 1 of the matheuristic, the number of feasible tours ($|FT|$) varies between 92 and 1470, with computational times (tcpu) varying from 26 seconds to less than 2 hours (see Table 2).

Instance	Matheuristic – step 1	
	$|FT|$	tcpu (s)
ex1	96	25.84
ex2	122	69.30
ex3	155	130.78
ex4	92	77.72
ex5	106	112.05
ex6	270	334.71
ex7	434	525.63
ex8	392	837.75
ex9	414	925.03
ex10	590	1458.09
ex11	1470	6378.74
ex12	1019	4367.67

Table 2: feasible tours generation.

Table 3 allows the comparison between the valid model, (M1DAR), and the matheuristic using model (MRHμ) with $\mu = 0$, named as (MRH0). As referred to, the similarity is treated in the same way in both models, so this is a fair comparison.

Second to fourth columns in Table 3 display the results for the valid model, (M1DAR), namely, the lower (LB) and upper (UB) total routing times provided by CPLEX and the computational time (tcpu) in seconds. Values obtained with the matheuristic using model (MRHμ), with $\mu = 0$, are in columns five and six. Column five, headed as GapUB0, depict gap values comparing upper bounds obtained by both models. Thus, if UB and $UB0$ are, respectively, the upper bounds for models (M1DAR) and (MRH0), $gapUB0 = \frac{UB0-UB}{UB} \times 100\%$. Thus, positive values represent instances for which (M1DAR) provides better bounds, while negative values point to a better performance of (MRH0).

Most of the times (M1DAR) succeeds in generating an optimal solution. In fact, this is always the case whenever $|H| = 3$, however for higher time horizon values its performance tends to decrease ($|H| = 4$: 8 optimums out of 12; $|H| = 5$: 7 out of 12). As stressed before, whenever (M1DAR) achieves an optimal solution, values in column five represent gap values between the heuristic upper bound and the optimum, and thus we may conclude that in 20 out of 36 instances (MRH0) ends up with an optimal solution. The biggest gap found was 5.2%, and (MRH0) got three better solutions than (M1DAR) during its three hours' cpu time limit. Moreover, this occurs for the bigger instances. Note that matheuristic cpu times, usually smaller than one minute and never greater than 12 minutes, can be considered reasonable.

Instance	(M1DAR)			Matheuristic & (MRH0)	
	LB	UB	tcpu (s)	GapUB0 (%)	tcpu (s)
			$\lvert H \rvert = 3$		
ex1	519*	519*	65.23	5.2	0.41
ex2	2200*	2200*	44.76	0.0	0.60
ex3	755*	755*	6.30	0.0	1.93
ex4	855*	855*	10.69	0.0	0.73
ex5	1161*	1161*	12.37	0.0	0.88
ex6	1110*	1110*	400.41	0.0	5.81
ex7	13700*	13700*	708.94	0.0	16.20
ex8	23148*	23148*	472.32	0.0	13.88
ex9	24935*	24935*	1221.27	0.0	13.94
ex10	2094*	2094*	4685.33♦	3.0	29.72
ex11	31395*	31395*	3350.59	0.3	203.39
ex12	38052*	38052*	7264.62♦	0.0	81.78
			$\lvert H \rvert = 4$		
ex1	704*	704*	830.84	3.7	0.59
ex2	2940*	2940*	279.66	0.0	0.94
ex3	1010*	1010*	148.07	0.0	4.09
ex4	1140*	1140*	47.41	0.0	1.16
ex5	1548*	1548*	125.50	0.0	1.32
ex6	**1468.72**	**1490**	10800.20	0.0	10.44
ex7	18310*	18310*	1090.94	0.0	28.94
ex8	30864*	30864*	964.19	0.1	26.44
ex9	33290*	33290*	8819.73♦	0.0	21.65
ex10	**2792**	**2837**	10801.20	2.8	58.21
ex11	**41860**	**42861**	10802.10	**-1.9**	446.47
ex12	**50732.3**	**50761**	10801.60	0.0	124.48
			$\lvert H \rvert = 5$		
ex1	871*	871*	522.58	4.6	0.85
ex2	3670*	3670*	377.74	0.0	1.34
ex3	1260*	1260*	883.27	0.0	4.37
ex4	1425*	1425*	211.24	0.0	1.77
ex5	1935*	1935*	16.37	0.0	1.79
ex6	**1827.97**	**1855**	10801.10	0.0	13.48
ex7	22855*	22855*	1793.69	0.0	40.29
ex8	38580*	38580*	2450.29	0.1	43.25
ex9	**41521**	**41580**	10803.30	0.0	30.06
ex10	**3490**	**3555**	10808.10	1.7	76.24
ex11	**52325**	**58917**	10812.70	**-10.9**	677.54
ex12	**63364.6**	**66175**	10802.30	**-4.2**	175.52

Table 3: computational results - (M1DAR) *vs* matheuristic with (MRH0).
Legend: * indicates the optimum; ♦ the optimum was reached in more than one hour of cpu time.

Henceforward, and for simplicity, model (MRHμ) for $\mu = 0$; 0.1; 0.3 is referred to as (MRH0), (MRH0.1) and (MRH0.3), respectively. Correspondent columns in the tables are headed by μ_0, μ_0.1 and μ_0.3. In relation to model (MRμ), results are presented for only $\mu = 0.3$, named as (MR0.3), with columns headed by μ_0.3.

Next, we compare the performance of the models used within the matheuristic, to select a vehicle service from the pool generated, FT. As referred to, (MRS) was used to compute μ=0.3 as the minimum value of μ to be applied in (MRμ). In fact, for smaller values no vehicle services can be found from the pool for too many instances (with $\mu = 0.3$ only instances ex3 and ex4 fail for $|H|$=5). Although it is assumed that an adequate way to deal with this real application is to limit the total similarity and then to minimise the routing time, it would be interesting to observe the total and the maximum similarities in the solutions obtained.

The bound on the total maximum similarity computed through propositions 1-3 is given in Table 4. Although not detailed in the tables, from the computational results we observed that while model (MRH0) almost always meet its maximum values, (MRH0.1) and (MRH0.3) are about 90% of its maximum, and (MR0.3) about 80%. We also noticed that the most frequent value regarding the maximum similarities of the feasible solutions for models (MRHμ) is one (the maximum), while (MRμ) is always close to 30%, which is its limit. Thus, models produce feasible solutions, within, as it is imposed, the upper similarity limits, and also very close to it.

TS upper bounb					
	(MRHμ)			(MRμ)	$C_2^{\|H\|}$
$\|H\|$	μ_0	μ_0.1	μ_0.3	μ_0.3	
$\|H\| = 3$	1	1.2	1.6	0.9	3
$\|H\| = 4$	2	2.4	3.2	1.8	6
$\|H\| = 5$	4	4.6	5.8	3.0	10

Table 4: upper bounds on TS values.

Table 5 depicts total routing time gap values for the matheuristic using models (MRH0), (MRH0.1), (MRH0.3) and (MR0.3) in columns two to five. Each instance bounds are computed against the better value. Thus, if RT^* is the lowest routing time value for an instance (i.e. the better upper bound) the percentage gap for model (MR#) that generates a feasible solution with a total routing time equal to $RT\#$ is $gap(MR\#) = \frac{RT\# - RT^*}{RT^*} \times 100\%$. Then, lines ending each group (for $|H| = 3,4,5$) display minimum, average and maximum gap values to summarize the information.

As can be observed, more restrictive models regarding similarity do not deteriorate to much the total routing time. In fact, models (MRH0) and (MRH0.1) routing time gap values vary between 0% and 9.3%, being, as expected, (MRH0.3) the better one. Model (MR0.3) sometimes fails or generate feasible vehicle services with very high total routing time. This results from the fact that $\mu = 0.3$, is the lowest value of the parameter that (MRμ) can handle, and probably FT includes only one group of $|H|$ tours that meet the similarity requirement, for smaller instances. However, for the bigger instances, its performance is quite good.

Routing time	(MRHμ)			(MRμ)
Instance	μ_0	μ_0.1	μ_0.3	μ_0.3
		$\|H\| = 3$		
ex1	9.0	2.8	0.0	6.0
ex2	0.5	0.5	0.0	0.5
ex3	0.7	0.7	0.0	0.0
ex4	0.0	0.0	0.0	0.0
ex5	0.0	0.0	0.0	0.0
ex6	1.4	1.4	0.0	0.0
ex7	0.5	0.3	0.0	0.3
ex8	0.0	0.0	0.0	0.0
ex9	0.3	0.0	0.0	0.0
ex10	3.0	0.0	0.0	0.0
ex11	0.3	0.0	0.0	0.0
ex12	0.0	0.0	0.0	0.0
Min	**0.0**	**0.0**	**0.0**	**0.0**
Average	**1.4**	**0.5**	**0.0**	**0.6**
Max	**9.0**	**2.8**	**0.0**	**6.0**
		$\|H\| = 4$		
ex1	9.3	4.2	0.0	12.1
ex2	0.7	0.7	0.0	0.7
ex3	1.0	1.0	0.0	0.5
ex4	0.0	0.0	0.0	0.0
ex5	0.0	0.0	0.0	0.7
ex6	2.1	2.1	0.0	2.1
ex7	0.7	0.4	0.0	0.6
ex8	0.1	0.0	0.0	0.0
ex9	0.4	0.0	0.0	0.2
ex10	4.4	0.0	0.0	0.5
ex11	0.4	0.0	0.0	0.0
ex12	0.0	0.0	0.0	0.0
Min	**0.0**	**0.0**	**0.0**	**0.0**
Average	**1.7**	**0.7**	**0.0**	**1.4**
Max	**9.3**	**4.2**	**0.0**	**12.1**
		$\|H\| = 5$		
ex1	9.1	3.4	0.0	7458.0
ex2	0.5	0.5	0.0	2663.7
ex3	0.8	0.8	0.0	0.8
ex4	0.0	0.0	0.0	-
ex5	0.0	0.0	0.0	-
ex6	1.6	1.6	0.0	15.1
ex7	0.6	0.3	0.0	0.7
ex8	0.1	0.0	0.0	0.1
ex9	0.3	0.0	0.0	0.3
ex10	3.6	0.0	0.0	1.8
ex11	0.3	0.0	0.0	0.0
ex12	0.0	0.0	0.0	0.0
Min	**0.0**	**0.0**	**0.0**	**0.1**
Average	**1.5**	**0.6**	**0.0**	**1014.0**
Max	**9.1**	**3.4**	**0.0**	**7458.0**

Table 5: total routing times comparing (MRHμ) and (MRμ).

Average, minimum and maximum values for the computational times (in seconds) referring to models (MRHμ) and (MRμ) may be consulted in Table 6. These values are considered small as they vary between 0 and 900 seconds (15 minutes).

tcpu	(MRHμ)			(MRμ)
	µ_0	µ_0.1	µ_0.3	µ_0.3
$\|H\| = 3$				
Min	0.41	0.27	0.22	0.02
Average	30.77	53.73	60.58	1.73
Max	203.39	344.50	385.48	11.33
$\|H\| = 4$				
Min	0.59	0.42	0.31	0.02
Average	60.39	86.10	97.03	1.54
Max	446.47	536.51	650.25	9.37
$\|H\| = 5$				
Min	0.85	0.58	0.45	0.02
Average	88.88	117.20	135.14	1.68
Max	677.54	734.49	894.37	11.21

Table 6: execution times comparing (MRHμ), (MRμ) and (MRS).

To sum up, for smaller instances the valid model (M1DAR) seems to be the best option. On the other hand, (MR0.3) seems to be a good option for larger instances, as having a maximum similarity for all pairs of tours controlled, its routing times are close to the better ones.

6. Final Remarks

In this work we present a new problem named DARP, Dissimilar Arc Routing Problem. It arises in one application where service is to be performed on arcs, every day of a time horizon, and similar tours should be avoided to prevent robberies.

We propose a definition of similarity between two tours based on the number of tasks that are visited by both tours in the same time periods of the day. Constraints can be used to prevent the selection of similar tours. A measure is also proposed to evaluate the total similarity of a group of tours.

An integer programing formulation is presented for DARP and the computational results show CPLEX is able to solve small sized instances. To deal with larger instances a matheuristic is developed. Framed on the matheuristic, different constraints to avoid similarity of tours were essayed and evaluated. One of the alternatives tested, (MR0.3), displayed a better balance for total routing time and total similarity, and it is not very demanding in terms of cpu time.

Topics for future research include several vehicles with a fixed capacity, different tasks demand for service in different days with distinct periodicities, as well as a further study on constraints to avoid similarities.

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Appendix: Flow based MCARP model

Adapting (Gouveia, et al., 2010), we may consider all tasks with unitary demands, a null dump cost, and each vehicle as a day that is used, and so for each day $h \in H$:

- $x_{ij}^h = \begin{cases} 1 & \text{if } (i,j) \in R \text{ is served in day } h \\ 0 & \text{otherwise} \end{cases}$
- y_{ij}^h is the number of times that arc $(i,j) \in A$ is deadheaded during day h.
- f_{ij}^h is the flow traversing arc $(i,j) \in A$ in day h. It is related to the remaining services in the tour, or in a subtour of it.

The problem to identify a vehicle service in H, minimizing the total routing time, is next detailed.

(MCARP)

$$\min Z = \sum_{h \in H} \left(\sum_{a \in A} d_a \; y_a^h + \sum_{a \in R} c_a \; x_a^h \right) \tag{48}$$

$$\sum_{j:(i,j) \in A} y_{ij}^h + \sum_{j:(i,j) \in R} x_{ij}^h - \sum_{j:(j,i) \in A} y_{ji}^h - \sum_{j:(j,i) \in R} x_{ji}^h = 0 \qquad i \in N \backslash \{0\}; h \in H \tag{49}$$

$$\sum_{j:(0,j) \in A} y_{0j}^h = 1 \qquad h \in H \tag{50}$$

$$\sum_{h \in H} x_{ij}^h = 1 \qquad a = (i,j) \in A_R \tag{51}$$

$$\sum_{h \in H} \left(x_{ij}^h + x_{ji}^h \right) = 1 \qquad a = (i,j) \in E_R \tag{52}$$

$$\sum_{j:(j,i) \in A} f_{ji}^h - \sum_{j:(i,j) \in A} f_{ij}^h = \sum_{j:(j,i) \in R} x_{ji}^h \qquad i \in N \backslash \{0\}; h \in H \tag{53}$$

$$\sum_{j:(0,j) \in A} f_{0j}^h = \sum_{a \in R} x_a^h \qquad h \in H \tag{54}$$

$$x_a^h \le f_a^h \le W \left(y_a^h + x_a^h \right) \qquad a \in R\,; h \in H;\ \ell \in L \tag{55}$$

$$f_a^h \le W y_a^h \qquad a \in A \backslash R\,; h \in H \tag{56}$$

$$x_{ij}^h \in \{0,1\} \qquad (i,j) \in R; h \in H; \ell \in L \tag{57}$$

$$f_{ij}^h \ge 0 \qquad (i,j) \in A; h \in H \tag{58}$$

$$y_{ij}^h \ge 0, \text{ integer} \qquad (i,j) \in A; h \in H; \ell \in L \tag{59}$$