SEMINÁRIO

ANÁLISE E EQUAÇÕES DIFERENCIAIS

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Some inequalities for fractional Laplacians

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Abstract:

Let Ω be a bounded domain with smooth boundary. We compare two natural types of fractional Laplacians $(-\Delta)^s$, namely, the "Navier" and the "Dirichlet" ones. We denote their quadratic forms by $Q^N_{s,\Omega}$ and $Q^D_{s,\Omega}$, respectively.

Theorem 1. Let $s > -1$, $s \notin \mathbb{N}_0$. Then for $u \in \mathrm{Dom}(Q^D_{s,\Omega})$, $u \not\equiv 0$, the following relations hold:

$$Q^N_{s,\Omega}[u] > Q^D_{s,\Omega}[u], \quad \text{if} \quad 2k < s < 2k+1, \quad k \in \mathbb{N}_0;$$
$$Q^N_{s,\Omega}[u] < Q^D_{s,\Omega}[u], \quad \text{if} \quad 2k-1 < s < 2k, \quad k \in \mathbb{N}_0.$$

Theorem 2. Let $0 < |s| < 1$, and let $u \in \mathrm{Dom}(Q^D_{s,\Omega})$, $u \geq 0$, $u \not\equiv 0$. Then the following relations hold (all inequalities are understood in the sense of distributions):

$$(-\Delta_\Omega)^s_N u > (-\Delta_\Omega)^s_D u, \quad \text{if} \quad 0 < s < 1;$$
$$(-\Delta_\Omega)^s_N u < (-\Delta_\Omega)^s_D u, \quad \text{if} \quad -1 < s < 0.$$

Theorem 3. For sign-changing $u \in \mathrm{Dom}(Q^D_{s,\Omega})$, the following relations hold:

$$Q^N_{s,\Omega}[u] > Q^N_{s,\Omega}[|u|]; \qquad Q^D_{s,\Omega}[u] > Q^D_{s,\Omega}[|u|], \quad \text{if} \quad 0 < s < 1;$$
$$Q^D_{s,\Omega}[u] < Q^D_{s,\Omega}[|u|], \quad \text{if} \quad 1 < s < 3/2.$$

This talk is based on joint papers with Roberta Musina, see [1]–[3].

[1] R. Musina, A.I. Nazarov, On fractional Laplacians // Comm. in PDEs, **39** (2014), N9, 1780–1790.

[2] R. Musina, A.I. Nazarov, On fractional Laplacians–2 // AIHP–AN. **33** (2016), N6, 1667–1673.

[3] R. Musina, A.I. Nazarov, A note on truncations in fractional Sobolev spaces // Bull. Math. Sci. V. 9 (2019), N1, 1-7. DOI:10.1007/s13373-017-0107-8.

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