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# SEMINÁRIO DE LÓGICA MATEMÁTICA

**Dia 22 de Outubro (segunda-feira), sala 6.2.33 às 16:00**

## Zigzag and Fregean arithmetic

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### Abstract:

In Frege's logicism, numbers are logical objects in the sense that they are extensions of certain concepts. Frege's logical system is inconsistent, but Richard Heck showed that its restriction to predicative (second-order) quantification is consistent. This predicative fragment is, nevertheless, too weak to develop arithmetic. In this paper, I will consider an extension of Heck's system with impredicative quantifiers. In this extended system, both predicative and impredicative quantifiers co-exist but it is only permissible to take extensions of concepts formulated in the predicative fragment of the language. This system is consistent. Moreover, it proves the principle of reducibility applied to concepts true of only finitely many objects. With the aid of this form of reducibility, it is possible to develop arithmetic in a thorough Fregean way. Despite the success in developing arithmetic in a strict logicist manner, we manifest some doubts as to whether this success extends to set theory.

[1] R. Heck. "The consistency of predicative fragments of Frege's Grundgesetze der Arithmetik", *History and Philosophy of Logic* 17: 209–220, 1996.

[2] F. Ferreira, "Amending Frege's Grundgesetze der Arithmetik", *Synthese* 147: 3-19, 2005.

[3] F. Ferreira, "Zigzag and Fregean arithmetic". In: *The Philosophers and Mathematics*, H. Tahiri (ed.). *Logic, Epistemology, and the Unity of Science* 43, 81-100, 2018. Springer International

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