SEMINÁRIO DE LÓGICA MATEMÁTICA

Dia 25 de Março (segunda-feira), sala 6.2.33

16:00

Some arguments in proof mining

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Abstract:

Proof mining is the search of quantitative information from non-effective mathematical proofs [1]. It is known that, in general, it is not possible to extract such information directly for \Pi^0_3 statements. Instead one must translate those statements into what is called their metastable version [2]. A good understanding of these metastability properties is of great importance when carrying out a quantitative analysis, as \Pi^0_3 statements do frequently appear in ordinary mathematics (e.g. in convergence or Cauchy properties).The purpose of this talk is to give a small insight of the inner works of proof mining and a theoretical explanation of how common mathematical arguments are analysed. Although these results are in essence true despite the technique used in proof mining, we will be focusing on the application of the bounded functional interpretation [3]. This work follows from recent developments in the application of this technique to the proof mining program [4][5][6][7]. We will look in some detail at the metastability property and explain how to give a quantitative meaning to two metastability properties taken simultaneously. Additionally, we will explain how to carry out an analysis of a mathematical proof that follows a discussion by cases. This helps in better understanding the difference between postulates (axioms added to an appropriate formal theory that serves as a background to the application of the proof-theoretical techniques) and implicative assumptions (antecedents of implications).

References:
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[2] Tao, T., Soft analysis, hard analysis, and the finite convergence principle. Essay posted May 23, 2007. Available at: http://terrytao.wordpress.com/2007/05/23/soft-analysishard-analysis-and-the-finite-convergence-principle/ (32, 37, 464)
[3] Ferreira, F. (2009). Injecting uniformities into Peano arithmetic. *Annals of Pure and Applied Logic*, *157*(2-3), 122-129.
[4] Ferreira, F., Leustean, L., & Pinto, P. (2018). On the removal of weak compactness arguments in proof mining. *arXiv preprint arXiv:1810.01508*.
[5] Leustean, L., & Pinto, P. (2019). Quantitative results on Halpern type proximal point algorithms (in preparation)
[6] Pinto, P. (2019). Metastability bounds on the Halpern type proximal point algorithm (in preparation)
[7] Dinis, B., & Pinto, P. (2019). Metastability of the proximal point algorithm with multi-parameters (in preparation)

Seminário financiado por Fundos Nacionais através da FCT – Fundação para a Ciência e a Tecnologia no âmbito do projeto UID/MAT/04561/2019

Local: FCUL, Edf. C6, sala 6.2.33