

# SEMINÁRIO LÓGICA MATEMÁTICA

**9 de Março | 16h00 | sala 6.2.33**

## Bealer's Intensional Logic (Part I)

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### **Abstract:**

Many intuitively valid arguments involving intensionality cannot be captured by first-order logic, even when extended by modal and epistemic operators.

Indeed, previous attempts at providing an adequate treatment of the phenomenon of intensionality in logic and language, such as those of Frege, Church, Russell, Carnap, Quine, Montague and others are fraught with numerous philosophical and technical difficulties and shortcomings.

We present Bealer's solution to this problem which hinges on an ontological commitment to theory of Properties, Propositions and Relations (PRP). At the most basic level we can distinguish two conceptions in the theory of PRPs. An objective one tied to modality and necessary equivalence, and a mental (intentional) one tied to concepts and the requirement of non-circularity in definitions. Building on the work of Russell, Church and Quine, Bealer proposes two distinct intensional logics T1 and T2 (presented in Hilbert form) corresponding to these two conceptions, both based on the language of first-order logic extended with an intensional abstraction operator.

In T1 necessitation can be directly defined and the axioms express S5 modal logic.

Both logics have a series of desirable features which set them apart from higher-order approaches. Bealer constructs a non-Tarskian algebraic semantic framework, distinct from possible worlds semantics, yielding two classes of models for which T1 and T2 are both sound and complete.

Other features include being able to deal with quantifying-in, and the various substitution puzzles, being free from artificial type restrictions, having a Russellian semantics, satisfying Davidson's learnability requirement, etc. Bealer unifies both logics to serve as a basis of a larger philosophical project in the tradition of logicism (or logical realism) as detailed in his book *Quality and Concept* (1982). This includes a neo-Fregean account of Arithmetic and Set Theory in which various purely logical (according to him) predication axioms (and intensional analogues of ZF, NGB, or Kelley-Morse axioms) are adjoined to T2, thereby explaining incompleteness as a property of pure logic rather than of mathematics. Surprisingly, and rather ironically, Bealer's logic also fulfils Carnap's thesis of extensionality due precisely to its ontological commitment to the reality of PRPs. In this series of two talks we will focus on the technical details of the proof of soundness and completeness of T1 and T2 and hint at some proof-theoretic and foundational developments.

